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STELLAR PARALLAX.

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(1). THE annual parallax of a star is the angle at the star subtended by the mean distance of the Earth from the Sun.

Imagine a right line drawn from the Earth to a star. As the Earth moves around the Sun this line will generate the surface of a cone having an elliptical base, and an observer on the Earth will always see the star on the surface of this cone. Generally therefore the small annual parallax will cause the star to describe an ellipse on the surface of the heavens. At the pole of the ecliptic this ellipse will be similar to the orbit of the Earth, while if the star is in the plane of the ecliptic it will move backward and forward in a right line. Since the Earth's orbit is nearly circular we see, from geometrical considerations, that the maximum value of the parallax of the star in longitude will occur when the longitudes of the earth and the star differ by a quadrant, and the maximum value of the parallax in latitude will occur when the star is in opposition to the Sun.

Let $\varDelta$ be the distance of the star from the Sun, and denote by λ and β its longitude and latitude when seen from the Sun. If we take the plane of the ecliptic as the plane of xy the values of the rectangular coordinates are

$$x = \varDelta \cos\beta \cos\lambda : \qquad y = \varDelta \cos\beta \sin\lambda : \qquad z = \varDelta \sin\beta.$$

Denoting by $\varDelta'$, λ', β' the corresponding quantities seen from the Earth we have

$$x' = \varDelta' \cos\beta' \cos\lambda' : \qquad y' = \varDelta' \cos\beta' \sin\lambda' : \qquad z' = \varDelta' \sin\beta'.$$

The ecliptic coordinates of the Sun referred to the Earth are

$$X = R\cos\Theta : \qquad Y = R\sin\Theta : \qquad Z = 0,$$

where R is the radius vector of the Earth, and Θ is the longitude of the Sun seen from the Earth. We have therefore the equations

$$\left.\begin{aligned} \Delta'\cos\beta'\cos\lambda' &= \Delta\cos\beta\cos\lambda + R\cos\Theta, \\ \Delta'\cos\beta'\sin\lambda' &= \Delta\cos\beta\sin\lambda + R\sin\Theta, \\ \Delta'\sin\beta' &= \Delta\sin\beta. \end{aligned}\right\} \qquad (1)$$

If we multiply the first of these equations by sin λ, the second by cos λ, and subtract the products we shall have

$$\Delta'\cos\beta'\sin(\lambda'-\lambda) = -R\sin(\lambda-\Theta).$$

In the present case we have $\Delta = \Delta'$, and $\beta = \beta'$, very nearly, and $\lambda'-\lambda$ is so small that we may put the arc for its sine. Also if we take the distance of the Earth from the Sun as our unit and denote the annual parallax of the star by π we have,

$$\sin\pi = \frac{1}{\Delta},$$

or in seconds of arc, since the parallax is small,

$$\pi = \frac{206264.8}{\Delta}.$$

Hence we have for the parallax in longitude,

$$\lambda'-\lambda = -\pi.R\sin(\lambda-\Theta)\sec\beta.$$

Again if we multiply the first of equations (1) by cos λ', the second by sin λ' and add the products we have

$$\Delta'\cos\beta' = \Delta\cos\beta\cos(\lambda'-\lambda) + R\cos(\lambda'-\Theta).$$

If now we put $\cos(\lambda'-\lambda) = 1$, we have from this equation and the third of equations (1)

$$\begin{aligned} \Delta'\cos\beta' &= \Delta\cos\beta + R\cos(\lambda'-\Theta), \\ \Delta'\sin\beta' &= \Delta\sin\beta. \end{aligned}$$

Multiplying the first of these equations by sin β, the second by cos β and subtracting the products we have as before

$$\beta'-\beta = -\pi.R\cos(\lambda-\Theta)\sin\beta.$$

Hence the formulæ for the parallax of a star in longitude and latitude are

$$\left.\begin{aligned} d\lambda &= -\pi.R\sin(\lambda-\Theta)\sec\beta, \\ d\beta &= -\pi.R\cos(\lambda-\Theta)\sin\beta. \end{aligned}\right\} \qquad (2)$$

(2). Since at the present time in all astronomical observations the plane of the equator is taken as the plane of reference, or as that of xy, we need expressions for the parallax in right ascension and declination. We may obtain these by means of equations (2) and the differential formulæ between the right ascension and declination and the longitude and latitude, but it is better to start directly from the fundamental equations.

Let α, δ, α', δ' be the right ascensions and declinations of the star seen from the Sun and the Earth, and denote by ε the obliquity of the ecliptic, the values of the coordinates referred to the plane of the equator are

$$\begin{array}{lll} x = \varDelta \cos\delta \cos\alpha, & y = \varDelta \cos\delta \sin\alpha, & z = \varDelta \sin\delta, \\ x' = \varDelta' \cos\delta' \cos\alpha', & y' = \varDelta' \cos\delta' \sin\alpha', & z' = \varDelta' \sin\delta', \\ X = R \cos\Theta, & Y = R \sin\Theta \cos\varepsilon, & Z = R \sin\Theta \sin\varepsilon. \end{array}$$

Hence we have the equations,

$$\left.\begin{array}{lll} \varDelta' \cos\delta' \cos\alpha' & = \varDelta \cos\delta \cos\alpha & + R \cos\Theta, \\ \varDelta' \cos\delta' \sin\alpha' & = \varDelta \cos\delta \sin\alpha & + R \sin\Theta \cos\varepsilon, \\ \varDelta' \sin\delta' & = \varDelta \sin\delta & + R \sin\Theta \sin\varepsilon. \end{array}\right\} \qquad (3)$$

Multiplying the first of these equations by $\sin\alpha$, the second by $\cos\alpha$, and subtracting the products we have

$$\varDelta' \cos\delta' \sin(\alpha' - \alpha) = R.(\sin\Theta \cos\varepsilon \cos\alpha - \cos\Theta \sin\alpha).$$

Multiplying the first of equations (3) by $\cos\alpha'$, the second by $\sin\alpha'$ and adding the products we have, if we put $\cos(\alpha' - \alpha) = 1$,

$$\varDelta' \cos\delta' = \varDelta \cos\delta + R.(\cos\Theta \cos\alpha' + \sin\Theta \cos\varepsilon \sin\alpha').$$

If we multiply this equation by $\sin\delta$, and the third equation of (3) by $\cos\delta$, and subtract the products, we shall have

$$\varDelta' \sin(\delta' - \delta) = R.\sin\Theta \sin\varepsilon \cos\delta - R.(\cos\Theta \cos\alpha' \sin\delta + \sin\Theta \cos\varepsilon \sin\alpha' \sin\delta).$$

Hence writing these in the usual form, and putting α for α', on the right hand side of the last equation, we have

$$\left.\begin{array}{l} \alpha' - \alpha = -\pi.R.[\cos\Theta \sin\alpha - \sin\Theta \cos\varepsilon \cos\alpha] \sin\delta, \\ \delta' - \delta = -\pi.R.[\cos\varepsilon \sin\alpha \sin\delta - \sin\varepsilon \cos\delta] \sin\Theta - \pi.R.\cos\alpha \sin\delta \cos\Theta. \end{array}\right\} \quad (4)$$

If now we put

$$\begin{array}{rl} a \sin A = & -\cos\varepsilon \cos\alpha \\ a \cos A = & -\sin\alpha \\ b \sin B = & \cos\varepsilon \sin\alpha \sin\delta - \sin\varepsilon \cos\delta \\ b \cos B = & -\cos\alpha \sin\delta, \end{array}$$

we have for the values of the parallaxes in right ascension and declination

$$\left.\begin{array}{l} d\alpha = \pi R.a \cos(\Theta + A) \sec\delta \\ d\delta = \pi R.b \cos(\Theta + B). \end{array}\right\} \qquad (5)$$

The above simple method of determining the parallax of a body by means of its rectangular coordinates referred to the different points appears to have been given first by Lagrange in his memoir on the transit of Venus, June 3, 1769. (Oeuvres de Lagrange, Tome II, p. 335.)

If we wish to observe a star which has a large parallax in right ascension we must find the maximum value of a in equations (5), and if we wish a large parallax in declination we have to find the maximum value of b in the same equations. From the values of $a \sin A$, $a \cos A$, $b \sin B$, $b \cos B$ we have

$$a^2 = 1 - \cos \alpha^2 \sin \varepsilon^2,$$

$$b^2 = 1 - (\cos \delta \cos \varepsilon + \sin \delta \sin \varepsilon \sin \alpha)^2,$$

and for the maximum values the last terms of these equations must be zero. Now from the spherical triangle between the pole of the equator, the pole of the ecliptic and the star, we have, if we denote by η the parallactic angle at the star,

$$\left.\begin{aligned} \cos \beta \sin \eta &= \cos \alpha \sin \varepsilon \\ \cos \beta \cos \eta &= \cos \delta \cos \varepsilon + \sin \delta \sin \varepsilon \sin \alpha. \end{aligned}\right\} \qquad (6)$$

These equations show that for the two maxima we have

$$\eta = 0; \qquad \text{and } \eta = 90^\circ:$$

or in the first case the circles of declination and latitude coincide, and in the second case they are perpendicular to each other. The coefficients a and b have simple geometrical meanings.

(3). The preceding formulæ (5) give the absolute parallax of a star in right ascension and declination, but the determination of the absolute position of a star is a difficult work, and generally in the case of stellar parallax it is better to proceed by differential methods. In this way the star is compared with one or two small stars near it, but which have no physical connection with it, and which may be assumed to be so much more distant that their parallaxes are insensible. This method gives us indeed only the relative parallax of the bright star, but it has the great advantage of permitting accurate differential observations, which will be very nearly independent of the position of the instrument. We may observe either differences of right ascension or of declination, or both, but generally it is better to observe the polar coordinates, that is the angle of position and the distance. To be adapted to this case the formulæ (5) need a transformation.

If we denote by s the distance, and by p the angle of position, and assume that s is so small that we may write the arc for its sine, we have

$$s \sin p = (\alpha - \alpha') \cos \delta$$
$$s \cos p = \delta - \delta',$$

and by differentiating

$$ds = \sin p \cos \delta . d\,(\alpha - \alpha') + \cos p . d\,(\delta - \delta')$$
$$s . dp = \cos p \cos \delta . d\,(\alpha - \alpha') - \sin p . d\,(\delta - \delta').$$

The values of the differentials $d(\alpha - \alpha')$ and $d\,(\delta - \delta')$ are the values of $-(\alpha' - \alpha)$ and $-(\delta' - \delta)$, given by equations (4). Substituting these values we have

$$ds = \pi R . \Big[\cos\theta \sin \alpha \sin p - \sin\theta \cos \varepsilon \cos \alpha \sin p + \cos \varepsilon \sin \alpha \sin \delta \sin \theta \cos p - \sin \varepsilon \cos \delta \sin\theta \cos p + \cos \theta \cos \alpha \sin \delta \cos p \Big],$$

$$dp = \frac{\pi R}{s} . \Big[\cos\theta \sin \alpha \cos p - \sin \theta \cos \varepsilon \cos \alpha \cos p - \cos \varepsilon \sin \alpha \sin \delta \sin\theta \sin p + \sin \varepsilon \cos \delta \sin\theta \sin p - \cos\theta \cos \alpha \sin \delta \sin p \Big].$$

If now we introduce the auxiliary quantities,

$$m \sin M = -\cos \varepsilon \cos \alpha \sin p + \cos \varepsilon \sin \alpha \sin \delta \cos p - \sin \varepsilon \cos \delta \cos p$$
$$m \cos M = \sin \delta \cos \alpha \cos p + \sin \alpha \sin p$$
$$m' \sin M' = \frac{1}{s} \Big[-\cos \varepsilon \cos \alpha \cos p - \cos \varepsilon \sin \alpha \sin \delta \sin p + \sin \varepsilon \cos \delta \sin p \Big]$$
$$m' \cos M' = \frac{1}{s} \Big[-\sin \delta \cos \alpha \sin p + \sin \alpha \cos p \Big],$$

we have the following formulæ for computing the effect of the parallax in distance and angle of position,

$$\left. \begin{aligned} ds &= \pi R . m \cos(\theta - M) \\ dp &= \pi R . m' \cos(\theta - M'). \end{aligned} \right\} \qquad (7)$$

These are the formulæ used by Bessel in his determination of the parallax of 61 Cygni. (Astronomische Nachrichten, Band 16, p. 83.)

The equations (6) give

$$\cos \beta \cos(\eta + p) = \cos \varepsilon \cos \delta \cos p + \sin \varepsilon \sin \delta \sin \alpha \cos p - \cos \alpha \sin \varepsilon \sin p,$$
$$\cos \beta \sin(\eta + p) = \cos \varepsilon \cos \delta \sin p + \sin \varepsilon \sin \delta \sin \alpha \sin p + \cos \alpha \sin \varepsilon \cos p.$$

If now we square and add the values of $m \sin M$, $m \cos M$, $m' \sin M'$, and $m' \cos M'$ we shall find

$$m^2 = 1 - \cos \beta^2 \cos(\eta + p)^2,$$
$$s^2\, m'^2 = 1 - \cos \beta^2 \sin(\eta + p)^2.$$

These equations show that the coefficients m and m' have their maximum values, and that the effect of the parallax on the distance and in the angle is greatest when the great circle joining the stars is perpendicular to the circle of latitude or coincides with it.

In order to render the formulæ for the auxiliaries m, m', M, M' convenient for calculation we may put with Brünnow,

$$f \sin F = \sin \delta \cos \alpha, \quad g \sin G = \sin \delta \sin \alpha, \quad h \sin H = g \sin(G + \varepsilon),$$
$$f \cos F = \sin \alpha, \quad g \cos G = -\cos \delta, \quad h \cos H = -\cos \alpha \cos \varepsilon,$$

and we shall have,

$$m \sin M = h \sin (H + p), \qquad m' \sin M' = \frac{h}{s} . \cos (H + p),$$

$$m \cos M = f \sin (F + p), \qquad m' \cos M' = \frac{f}{s} . \cos (F + p).$$

By means of these formulæ the coefficients are easily computed.

The observations should extend over a year, so that the coefficients of the parallax may attain their maximum and minimum values with opposite signs. The observations must be corrected for refraction, which in the case of differential observations is a simple work, and then reduced to a chosen epoch by applying the necessary corrections for precession, nutation, aberration and proper motion. This reduction is also easily made in the case of differential observations. The observations are then ready to form the equations of condition.

If we denote by p_0 and s_0 assumed values of the angle of position and distance at the epoch t_0; by dp_0 and ds_0 the corrections to these assumed values; by dp' and ds' the corrections to the assumed annual variations of p and s produced by the proper motion, and designate by a and b the coefficients for the parallax, each complete observation gives two equations of condition of the form,

$$dp_0 + (t - t_0).dp' + a\pi + n = 0,$$
$$ds_0 + (t - t_0).ds' + b\pi + n' = 0,$$

where n and n' are the residuals found by comparing the computed with the observed place. Professor Brünnow, to whom we owe the latest and some of the best determinations of stellar parallax, assumes further that the constant of aberration may be different for the two stars, and introduces into the equations a term to correct the assumed value of this constant. The coefficients for this term are easily computed, for we have only to put $\theta - 90°$, for θ, in the formulæ for the coefficients of the parallax.

(4). If we determine the parallax of a binary star whose orbit is known, and assume that the orbital motion of the stars in the distant spaces are governed by the law of gravitation, we shall have the data for a determination of the sum of the masses of the two stars. From the theory of elliptic motion we have the known equation,

$$\mu.\tau^2 = \left(\frac{a}{\pi}\right)^3 :$$

where μ is the sum of the masses of the stars, the mass of our Sun being the unit of mass, τ the time of their revolution, expressed in sidereal years, a the angle subtended by the semi-major axis of their orbit, and π the annual parallax. It is remarkable that Bessel in 1812, nearly thirty years before his determination of the parallax of 61 Cygni, estimated by means of the above equation the annual parallax of this star to be very nearly what the later determinations have given. Bessel, after his discussion of the proper motion of this star, assumed

$$a = 25''; \qquad \tau = 400 \text{ years},$$

and put $\mu =$ the mass of the Sun $= 1$.

The equation gives

$$\pi = \frac{a}{\tau^{\frac{2}{3}}\mu^{\frac{1}{3}}} = 0''.4604.$$

In 1840 Bessel found from his observations the value

$$\pi = 0''.3483 \pm 0''.0095.$$

Besides the preceding method of estimating the distances of the fixed stars by assuming a knowledge of their masses, Savary, in an interesting paper on the orbits of double stars, published in the *Connaissance des Tems* for 1830, has remarked that the aberration of light may give us some idea of these distances. If the plane of the orbit of the double star were perpendicular to the line of sight, aberration would produce only insensible changes in the motions; but if this plane be much inclined to the line of sight, the time which light requires to pass over the radius of the orbit will produce apparent variations in the angular movements of the stars, and if these could be determined by observation they would give the distance of the stars. In order to show the maximum effect of this variation let us suppose with Struve that the major axis of the orbit is directed toward the Sun, and that the star is at its perihelion. The time which light requires to pass over the axis of the orbit will be

$$z = 0.00003155.\frac{a}{\pi},$$

where the unit of z is one year. In this time the apparent angular motion of the double star seen from the Earth will be

$$\gamma = \frac{z}{\tau} \cdot \frac{3.1416 \cdot 2a}{1 - e},$$

where e is the excentricity of its orbit. From the value of z we have

$$\gamma = \frac{0.0001982 \cdot a^2}{\pi(1-e) \cdot \tau};$$

or since $a = \pi \cdot \tau^{2/3} \cdot \mu^{1/3}$; we have the ratio of γ and π,

$$\frac{\gamma}{\pi} = \frac{0.0001982 \cdot \tau^{1/3} \cdot \mu^{2/3}}{1 - e}.$$

This equation shows that if the masses of the stars are not much greater than the mass of our Sun then γ is much smaller than π, unless the periodic time is very great. Thus if we suppose the mass of the two stars to be one thousand times as great as the mass of our Sun, it will be seen that τ must still be very great in order to make γ as large as π. Moreover this inequality would probably be connected with the equation of centre in such a way as to make its determination nearly impossible. The idea of Savary therefore, although ingenious and founded on a real phenomenon, does not seem to promise any practical result.

(5). The only practicable method of determining the distance of a star appears to result from the determination of its annual parallax by observation, and I wish to call the attention of American astronomers to this interesting problem. The theory is simple, the reductions are easy, and the final calculation of the equations of condition and their solution by the method of least squares presents no great difficulties. In this problem the astronomer has the satisfaction of knowing that the value of the result lies wholly in his own hands. It will depend on the care and thoroughness with which he has studied and mastered his instrument, and on his skill in making the observations. As the quantity to be determined is small, and has a period of a year, it may easily be masked by the yearly change of temperature, or a habit of observing differently on different sides of the meridian, so that there is ample room for thoughtful observation. An equatorial telescope, having an objective of ten or twelve inches aperture, and provided with a good filar micrometer and a driving clock, is sufficient. The excellent objectives made by Alvan Clark and Sons are well suited to this work, since they give clear, sharp images of the stars. Many of these telescopes are now established in different parts of our country, and perhaps a few of them might be used for observations of stellar parallax.